Figure-8 Electron Ring – Small Equilibrium Emittance Lattice

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Abstract

To maintain high polarization of electrons in a collider ring it is advantageous to use a Figure-8 configuration rather than a conventional circular ring. Maximizing collider luminosity requires small equilibrium emittance governed by quantum excitations. This can be achieved through a careful lattice design, by appropriate 'tailoring' of the Twiss functions and their derivatives in the bending magnets. Three styles of low emittance optics were examined: the FODO, the Double Bend Achromat (DBA) and the Triple Bend Achromat (TBA). The 135 deg. phase advance FODO structure offers great lattice compactness, while the DBA and TBA based rings excel in minimizing the equilibrium emittance. For a compact collider ring with the RF confined to one or two long straights, the FODO based lattice seems most suitable. One can still maintain appropriately small equilibrium emittance driven by the collider luminosity, while taking advantage of uniform focusing and superior lattice compactness. Here we will present linear optics design for the Figure-8 lattice topology based on the 135 deg. FODO structure.

1. Natural Equilibrium Emittance

Synchrotron radiation effects are of paramount importance for the motion of electrons in a storage ring. Each time a quantum is emitted the energy of the electron suffers a small discontinuity. Sudden emissions of individual photons excite various oscillations; the resulting energy 'drop' disturbs the trajectory of the electron causing their amplitudes to grow. However, for the ultra-relativistic electrons the radiation is emitted primarily along the direction of motion within a narrow $1/\gamma$ cone, therefore the resulting momentum change is opposite to the direction of motion. This radiation reaction force is to be balanced by the action of the RF system.

In a storage ring the electron beam reaches the state of equilibrium when the quantum emission excitations of both transverse and longitudinal oscillations are balanced by the radiation damping originating from the action of the RF system. Because of the statistical nature of the quantum emission the equilibrium is characterized by a Gaussian distribution. Details of single particle dynamics were given by M. Sands; here are some major results [1]

Assuming the isomagnetic guide field, defined as follows:

$$\frac{1}{\rho(s)} = \frac{1}{\rho_0}, \qquad \text{inside the bending magnet}$$

$$\frac{1}{\rho(s)} = 0, \qquad \text{elsewhere,} \tag{1}$$

the natural beam emittance is given by the following expression

$$\varepsilon_x = \frac{C_q \left\langle H \right\rangle_{mag} \gamma^2}{J_x \rho_0}, \tag{2}$$

where
$$H(s) = \frac{1}{\beta(s)} \left\{ D^2(s) + \left[\beta(s) D^{'}(s) - \frac{1}{2} \beta^{'}(s) D(s) \right]^2 \right\}$$

and the following integral over all bending magnets is carried out:

$$\left\langle ... \right\rangle_{mag} = \frac{1}{2\pi\rho_0} \int_{mag} ds...$$

Here,

$C_q = 3.84 \times 10^{-13} [m]$ is the so called quantum constant

$J_x \approx 1$ is the damping partition number for synchrotron radiation.

The above expression, Eq.(2), will be used as a starting point to design appropriate optics for the Figure-8 collider ring, which will be discussed in the next few sections of this paper. In summary, the equilibrium emittance of the collider ring will be evaluated numerically for the final optics design; both using the analytic formula and the exact emittance dilution calculation via OptiM (beam optics code) [3].

2. Small Equilibrium Emittance Lattices

By careful lattice design one can appropriately 'tailor' Twiss functions and their derivatives in the bending magnets, so that the value of $\langle H \rangle_{mag}$ is minimized.

The H-function can be expressed analytically [2] for various types of lattices; then the equilibrium emittance can be written in the following compact form:

$$\varepsilon_x^{\min} = C_q \; k_i \; F_i(\mu_c) \; \frac{\phi^3 \gamma^2}{J_x} \; [m \; rad], \tag{3}$$

where $\phi = \frac{L}{\rho}$ [rad]

is a single dipole bend angle and the factors $k_i F_i(\mu_c)$ depend only on the type of lattice structure. Here we considered three styles of cells: the FODO, the Double Bend Achromat and the Triple Bend Achromat – the corresponding $k_i \; F_i(\mu_c)$ factors are summarized below [2]:

$k_{FODO} = 4$ $\quad F_{FODO}(3\pi/4) = 0.62$ $\quad k_{FODO} \; F_{FODO}(3\pi/4) = 2.48$

$k_{DBA} = \frac{1}{4\sqrt{15}}$ $\quad F_{DBA}(\mu_c) = 1$ $\quad k_{DBA} \; F_{DBA}(\mu_c) = 0.065$

$k_{TBA} = \frac{7}{36\sqrt{15}}$ $\quad F_{TBA}(\mu_c) = 1$ $\quad k_{DBA} \; F_{DBA}(\mu_c) = 0.050$

As shown in [2] for the FODO optics, the above F-factor depends on the phase advance per cell, μ_c having a shalow minimum at $3\pi/4$ (135 deg.) [4].

All three styles of low emittance cells (based on the same bend angle magnet) are illustrated in terms of Twiss functions in Figure 1.

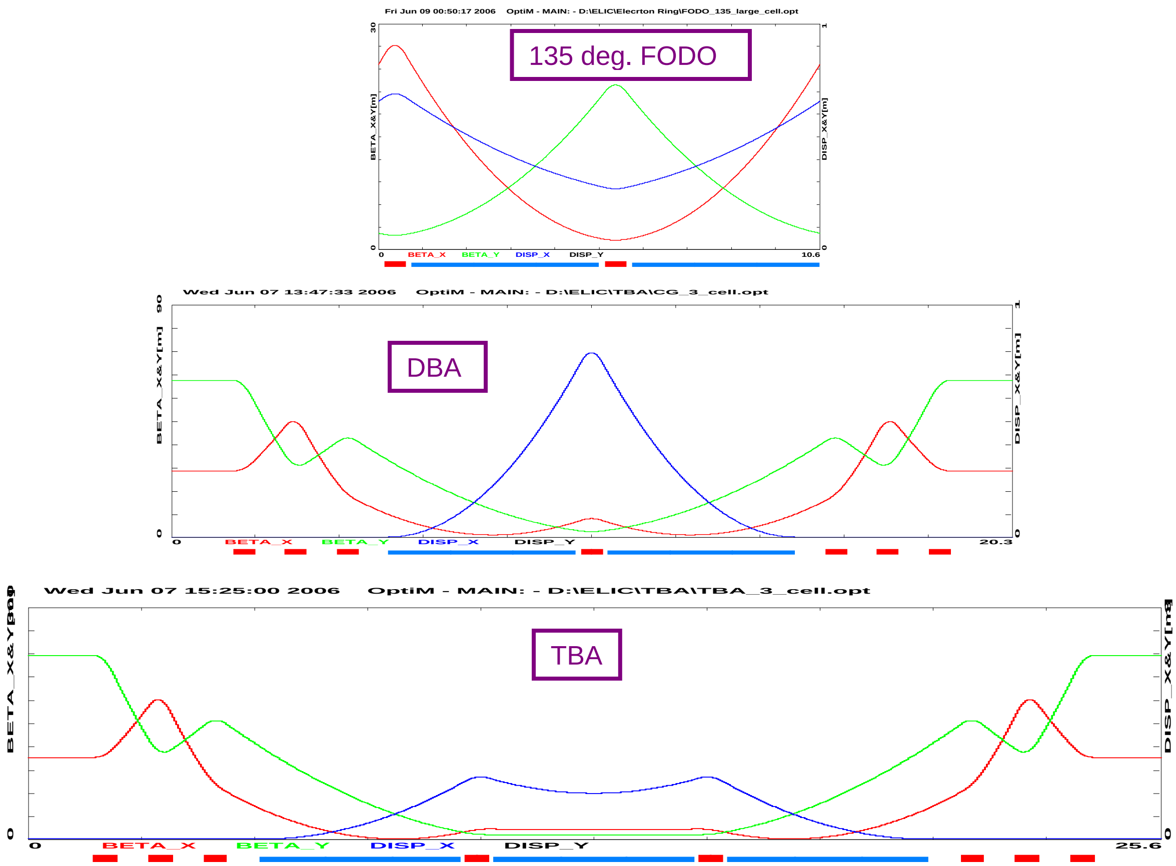

Figure 1 Low equilibrium emittance lattices: FODO, DBA and TBA periodic cells

As one can see from Figure 1, the FODO structure offers great lattice compactness compare to the DBA and TBA cells (roughly factor of 2 longer then the FODO), while the DBA and TBA based rings excel in minimizing the equilibrium emittance (about factor of 40 down from the FODO). Naturally, one would use the achromat cells (DBA or TBA) to build a high brilliance synchrotron light source where there is a great need for even distribution of the RF throughout the ring, since each cell offers a dispersion free straight suitable to host RF cavities. On the other hand, for a compact collider ring with the RF confined to one or two long straights, the FODO based lattice seems more suitable. One can still maintain appropriately small equilibrium emittance driven by the collider luminosity consideration while taking advantage of uniform focusing and superior lattice compactness.

3. Figure-8 Collider Ring Architecture

To maintain high polarization of the electron beam in a collider ring there is a great advantage of the Figure-8 configuration vs. a conventional 360 deg. ring. Here we will present linear optics design for such lattice topology based on the previously described 135 deg. FODO structure.

First, one needs to design an achromat super-period out of 135 deg. FODO cells. Starting with zero dispersion and its derivative at the beginning of the achromat one needs to advance the betatron phase by a multiple of 2π to create a periodic dispersion wave (zero dispersion and its derivative at the end). This can be accomplished by putting together minimum of eight 135 deg FODO cells as shown by a simple numerology: $8\times3\pi/4 = 3\times2\pi$. The resulting achromat super-period (a sequence of eight 135 deg. FODO cells) is illustrated in Figure 2.

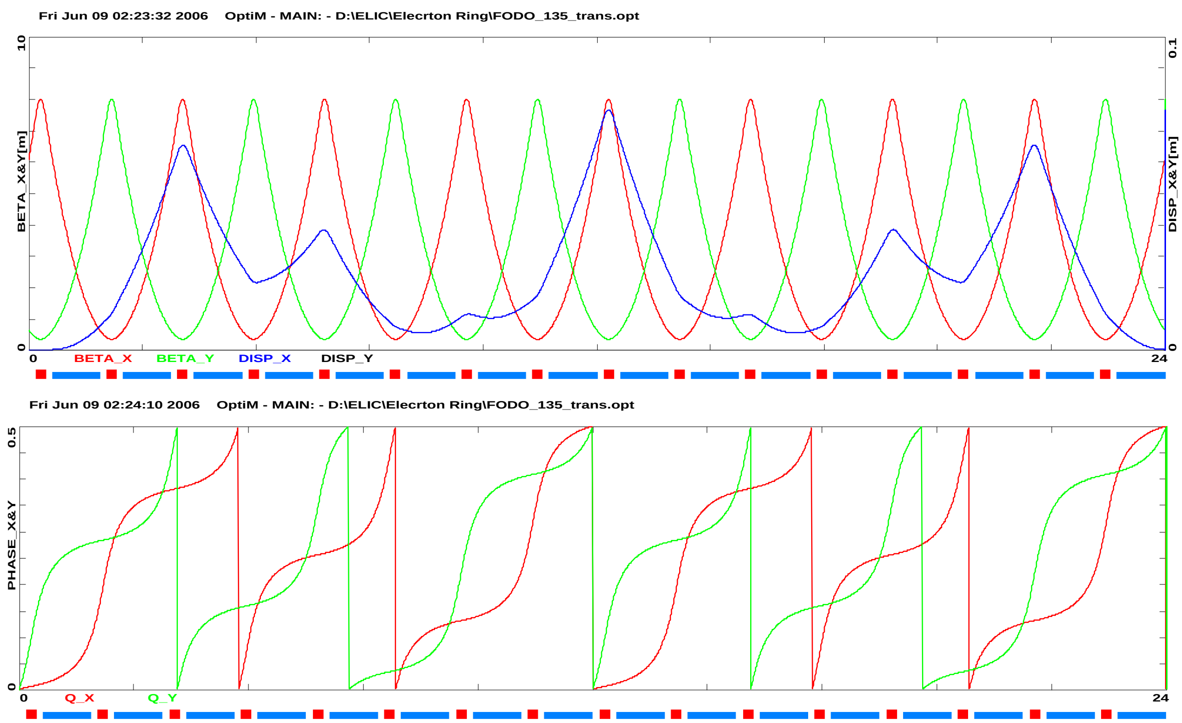

Figure 2 Achromat super-period – Twiss functions (top) and betatron phase advance in units of 2π (bottom)

The above periodic module will be used as a building block to construct bending parts 'loops' of the Figure-8 ring. The achromat super-period is also naturally matched to individual 135 deg. FODO cells with removed dipoles – the so called ‘empty’ cells. The empty cells will be used to construct the straight sections of the Figure-8 ring.

The overall optics for one half of the Figure-8 ring (where 240 deg. bend is closed by ten super-periods) at 7 GeV is illustrated in Figure 3. Its geometric layout is depicted in Figure 4.

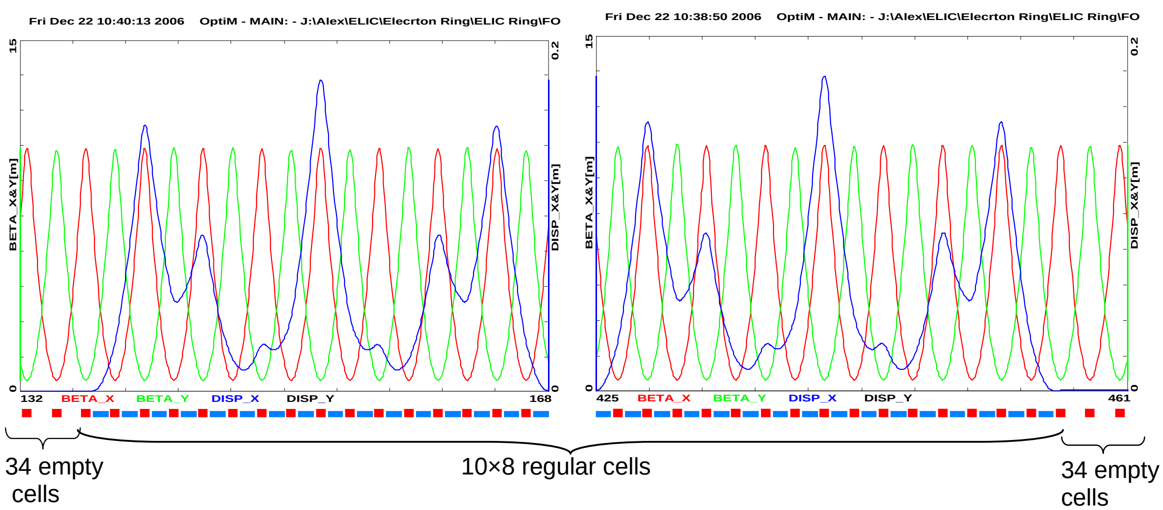

Figure 3 Linear optics for one half of the Figure-8 ring with 60 deg. crossing.

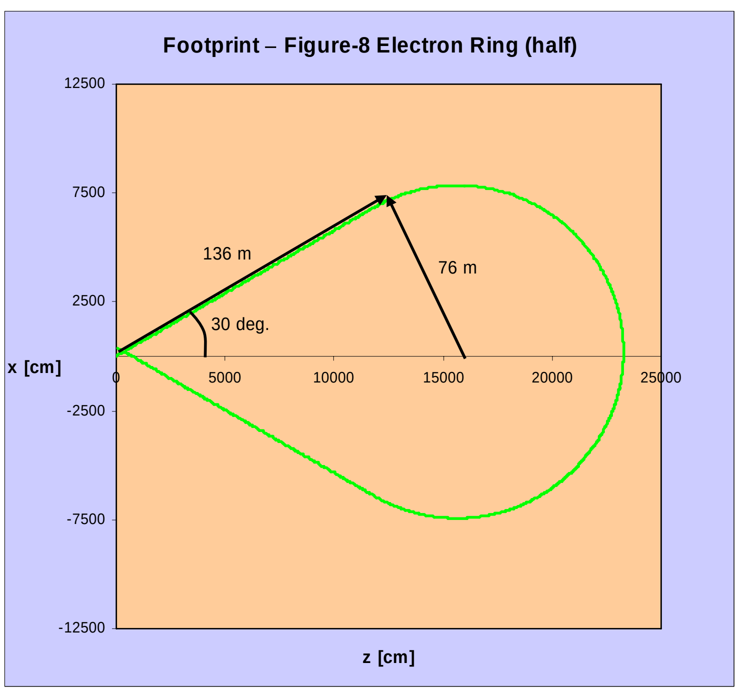

Figure 4 Layout of one half of the Figure-8 ring with 60 deg. crossing.

The long dispersion free straights (2×136 m each) will accommodate the RF and as many as four interaction regions (IR). The FODO structure of the straights is quite flexible to 'launch' matching inserts around the IRs.

4. Equilibrium Emittance of the Figure-8 Ring

The equilibrium emittance for the above Figure-8 lattice can be evaluated numerically from Eq. (2) modified for the new 'topology' – full closing of the Figure-8 ring requires 480 deg. of net bending rather than usual 360 deg. in the conventional circular layout. The resulting modified formula acquires a factor of 4/3 (480/360) as expressed below

$$\varepsilon_x^{\min} = 9.52 \times 10^{-13} \ \frac{4}{3} \ \phi^3 \gamma^2 \ [m\ rad], \qquad (4)$$

The above formula was evaluated numerically for two lattice varieties fitting in the layout illustrated in Figure 4: the 'small emittance' lattice with fewer longer dipoles (240 deg loop closed with 9 super periods – total of 9×8×2 = 144 'long' dipoles) and the 'very small emittance' lattice with larger number of shorter dipoles (240 deg loop closed with 19 super periods – total of 19×8×2 = 304 'short' dipoles). Both results are summarized below including the equilibrium emittance evaluated from the lattice H-functions as calculated numerically by OptiM [3] :

Lattice variety	**'small' emit. lattice**	**'very small' emit. lattice**
number of bends	288	608
dipole bend angle [mrad]	29.08	13.77
dipole length [cm]	50	100
dipole field [kGauss]	6.44	6.79
equilibrium emittance (analytic) [nm rad]	5.87	0.623
equilibrium emittance (OptiM) [nm rad]	5.97	0.635

5. Summary

Here we considered the Figure-8 configuration of a collider ring, based on its suitability to maintain high polarization of the electron beam. Collider luminosity considerations require small equilibrium emittance, which can be achieved through a careful lattice design of the Twiss functions in the bends. Three styles of low emittance optics were examined: the FODO, the Double Bend Achromat (DBA) and the Triple Bend Achromat (TBA). The 135 deg. phase advance FODO structure offers uniform focusing and superior lattice compactness, while still maintaining appropriately small equilibrium emittance. Complete lattice design at 7GeV for the Figure-8 collider topology based on the 135 deg. FODO structure was presented. The key parameters of the Figure-8 ring, computed via OptiM [3], are summarized in the Table below:

Figure-8 Electron Ring – 'Small' Emittance Lattice	
circumference, C [m]	1184
arc bending radius, R [m]	76
dipole bending radius, ρ [m]	38
average betas (h/v) [m]	6.2/6.2
average dispersion, D_x [cm]	7
betatron tunes (h/v)	111.01/110.9
chromaticities (h/v)	-226.41/-226.04
$M_{56} = \int \frac{D_x}{\rho} ds$ [cm]	46
momentum compaction, $\alpha = M_{56}/C$	3.9×10^{-4}
transition gamma, $\alpha = \frac{1}{\gamma_t^2}$	51

References

1. M. Sands, The Physics of Electron Storage Rings, an Introduction, SLAC-121 (1970)

2. H. Wiedeman, Nucl. Instr. and Meth. **172** (1980) 33

3. http://www-bdnew.fnal.gov/pbar/organizationalchart/lebedev/OptiM/optim.htm

4. Andrew Hutton, private communication